The background of the slide is a light beige color with several realistic water droplets of various sizes scattered across it. The droplets have highlights and shadows, giving them a three-dimensional appearance. The title text is centered in the middle of the slide.

# TECNICAS DE PRONOSTICOS

M.Sc. Christian W. Barreto Schuler

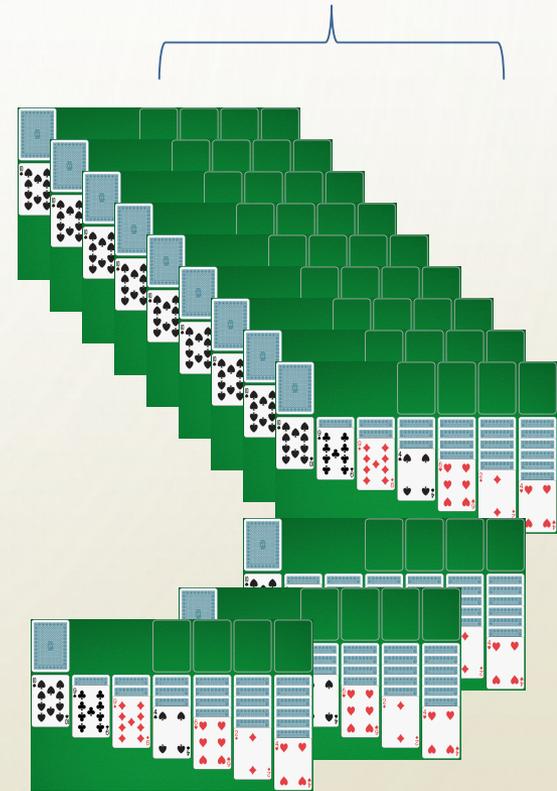
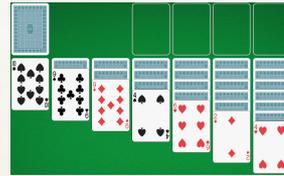
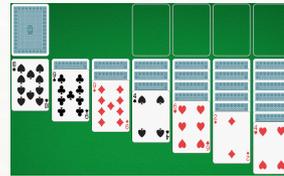
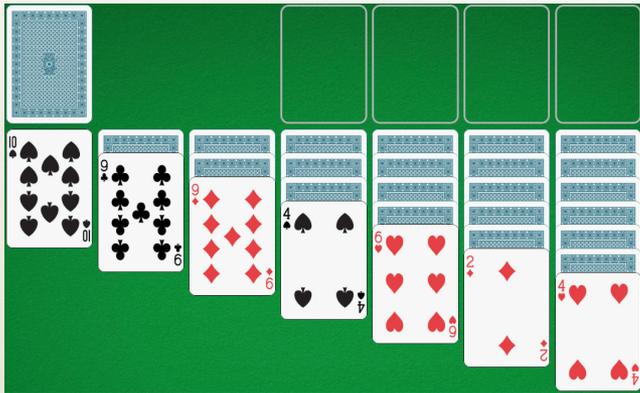
# CONTENIDO

- IV. ANALISIS EXPLORATORIO DE DATOS, DISTRIBUCIONES EMPIRICAS Y PARAMETRICAS

# MONTE CARLO

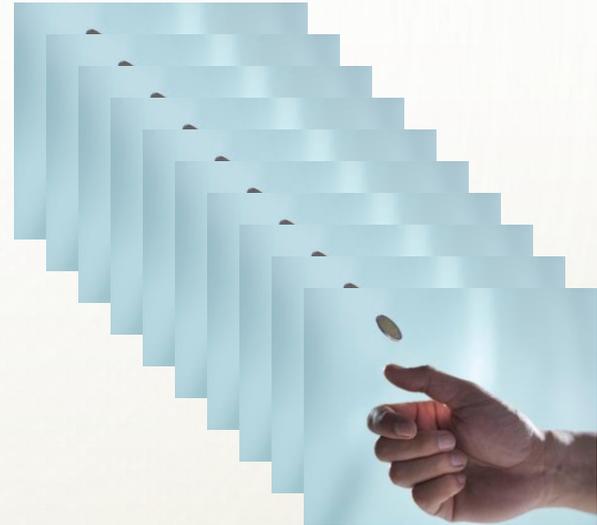
*Stanislaw Ulam*

- Random=Aleatorio



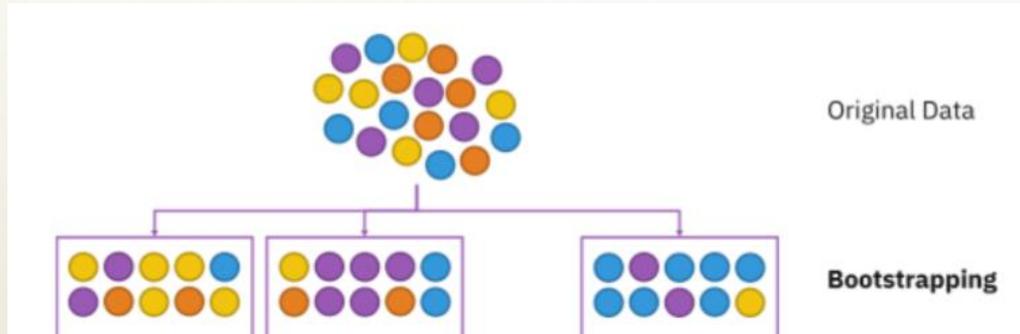
n veces

# MONTE CARLO



# BOOTSTRAPPING: INTERVALOS DE CONFIANZA

Muestreo con repetición

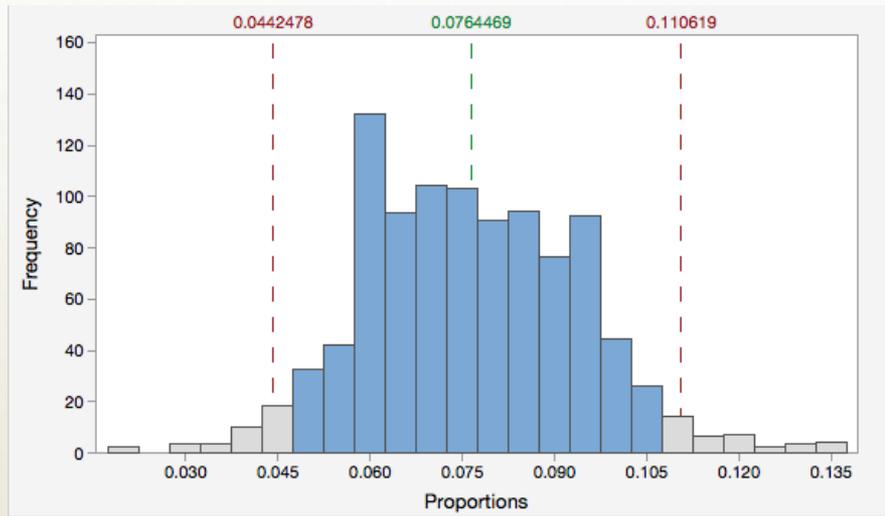


Joshi, 2020\*

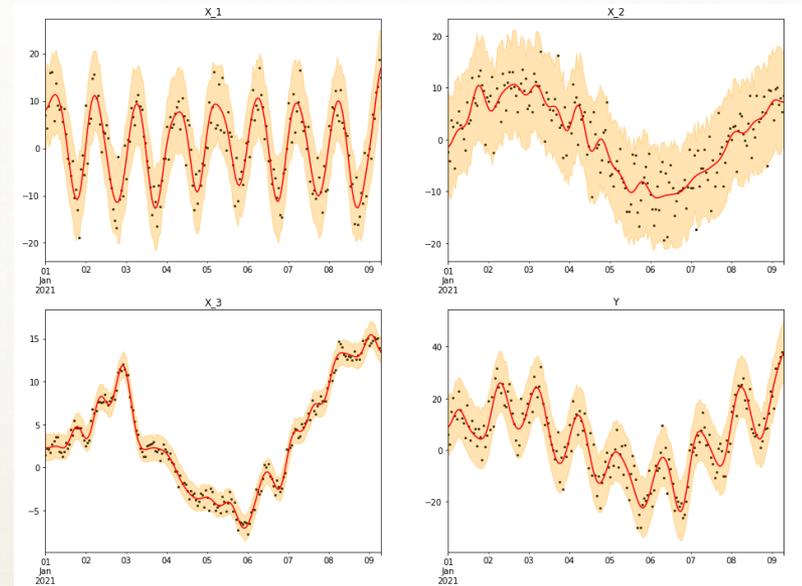


\*<https://www.analyticsvidhya.com/blog/2020/02/what-is-bootstrap-sampling-in-statistics-and-machine-learning/>

# BOOTSTRAPPING: INTERVALOS DE CONFIANZA



<https://online.stat.psu.edu/stat200/book/export/html/97>



<https://towardsdatascience.com/time-series-bootstrap-in-the-age-of-deep-learning-b98aa2aa32c4>

***PRACTICA***

# HISTOGRAMAS

- Es una manera grafica de representar la distribución de frecuencias de un conjunto de datos.
- Los datos suelen dividirse en intervalos, y el numero de los valores caen en cada intervalo.
- El histograma consiste en una serie de rectángulos donde sus alturas dependen de el numero de valores en cada intervalo.
- Histogramas, revelan rápidamente información sobre ubicación, dispersión, y simetría de los datos.
- Las áreas de las barras del histograma son proporcionales a probabilidades

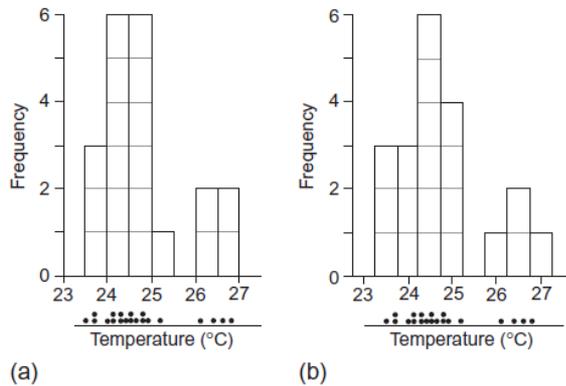


FIGURE 3.6 Histograms of the June Guayaquil temperature data in Table A.3, illustrating differences that can arise due to arbitrary shifts in the horizontal placement of the bins. Neither of these plots in panels (a) or (b) is more “correct” than the other. This figure also illustrates that each histogram bar can be viewed as being composed of stacked “building blocks” (gray) equal in number to the number of data values in the bin. Dotplots below each histogram locate the original data.

# HISTOGRAMAS

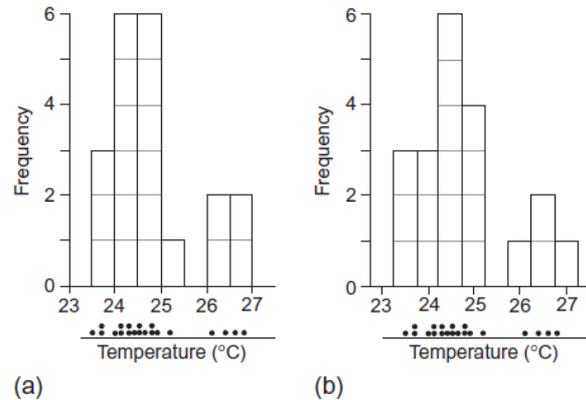


FIGURE 3.6 Histograms of the June Guayaquil temperature data in Table A.3, illustrating differences that can arise due to arbitrary shifts in the horizontal placement of the bins. Neither of these plots in panels (a) or (b) is more “correct” than the other. This figure also illustrates that each histogram bar can be viewed as being composed of stacked “building blocks” (gray) equal in number to the number of data values in the bin. Dotplots below each histogram locate the original data.

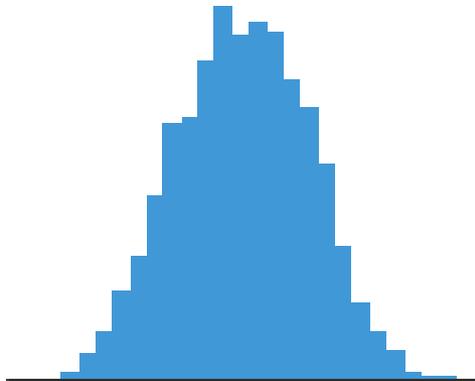
Wilks, 2019

El principal problema que se debe afrontar al construir un histograma es la elección del ancho de intervalo. Los intervalos que son demasiado amplios darán como resultado que se enmascaren detalles importantes de los datos. Los intervalos demasiado estrechos darán como resultado un gráfico irregular y difícil de interpretar.

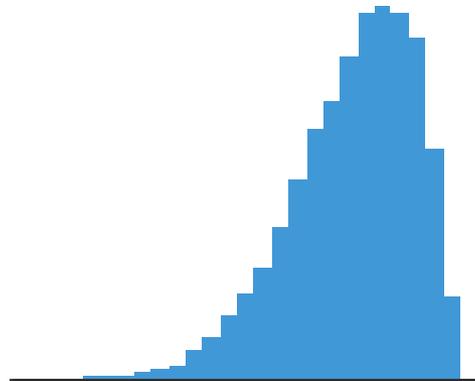
$$w \approx \frac{cIQR}{n^{1/3}},$$

$2 \leq c \leq 2.6$ ,  $c$  es una constante

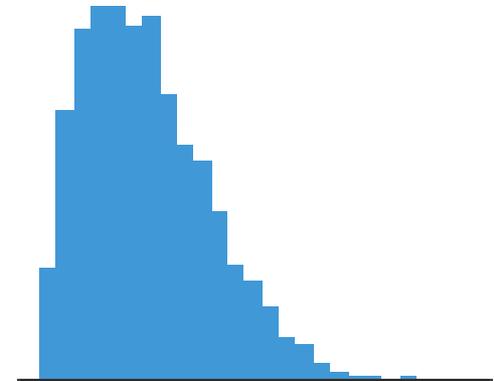
# HISTOGRAMAS



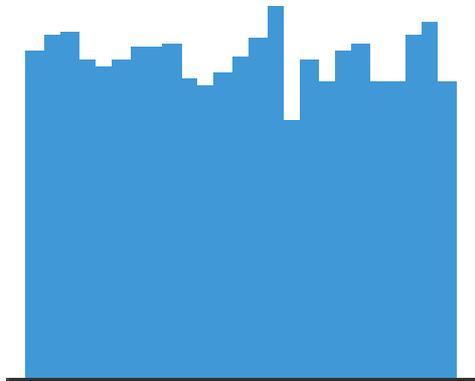
symmetric, unimodal



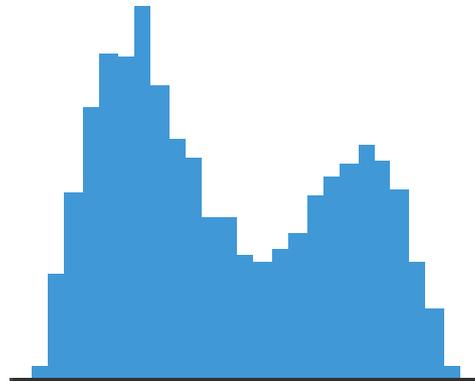
skew left  
Sesgado a la izquierda



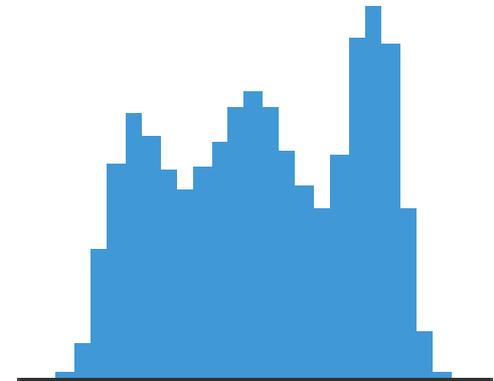
skew right  
Sesgado a la derecha



uniform



bimodal

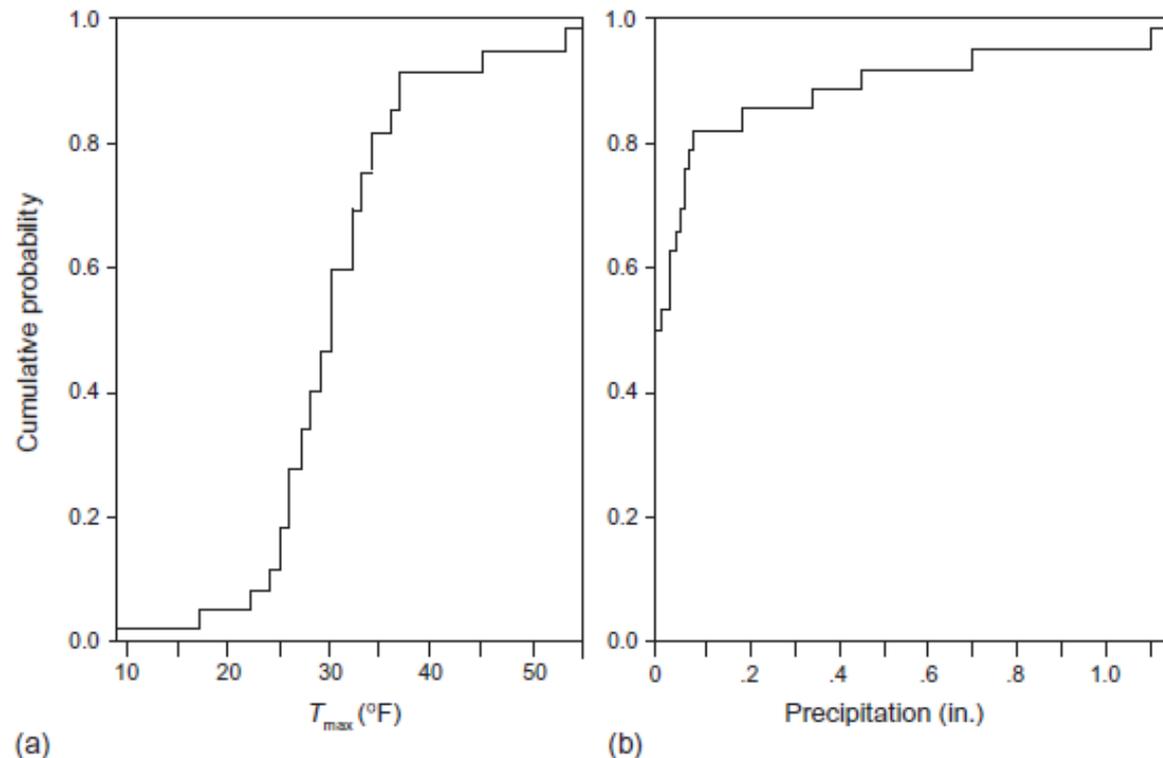


multimodal

Yi, 2019

# HISTOGRAMAS

## Distribucion de frecuencia acumulada



**FIGURE 3.11** Empirical cumulative frequency distribution functions for the January 1987 Ithaca maximum temperature (a), and precipitation data (b). The S-shape exhibited by the temperature data is characteristic of reasonably symmetrical data, and the concave downward character exhibited by the precipitation data is characteristic of data that are skewed to the right.

# PDF (PROBABILITY DENSITY FUNCTION)

Función de densidad; también llamada función de frecuencia.) Es la función estadística que muestra cómo se distribuye la densidad de posibles observaciones en una población.

La función de densidad no es negativa y su integral total es la unidad. A veces, la función de densidad de probabilidad se denomina función de distribución, pero esta práctica genera confusión y no se recomienda.

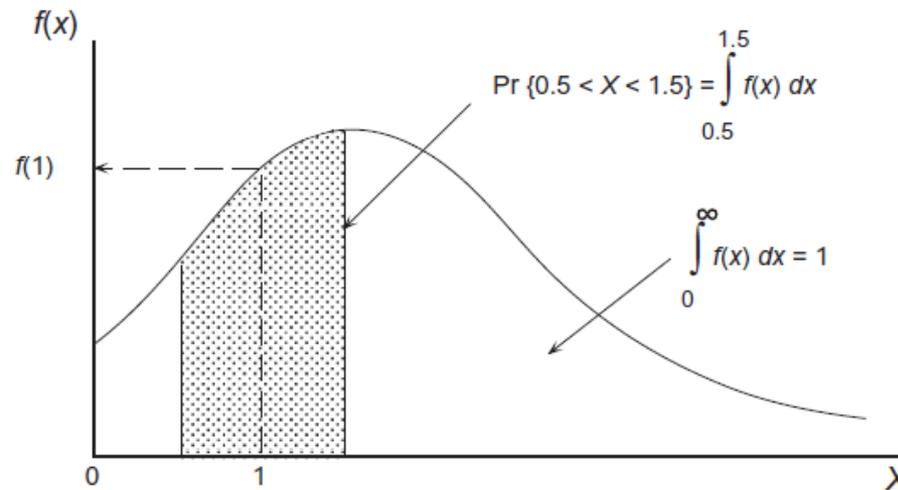
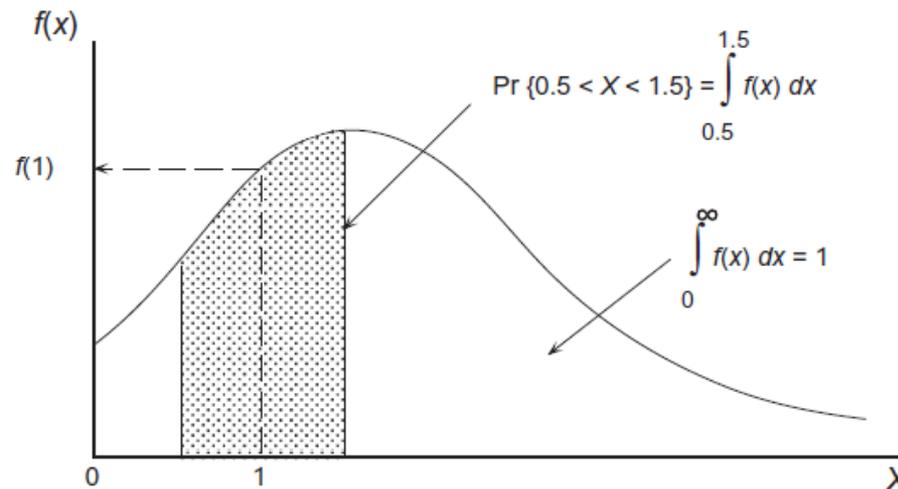


FIGURE 4.3 Hypothetical PDF  $f(x)$  for a nonnegative random variable,  $X$ . Evaluation of  $f(x)$  is not, by itself, meaningful in terms of probabilities for specific values of  $X$ . Probabilities are obtained by integrating portions of  $f(x)$ .

# PDF (PROBABILITY DENSITY FUNCTION)

La figura 4.3 muestra una PDF hipotética, definida para una variable aleatoria no negativa  $X$ . Una PDF puede evaluarse para un valor específico de la variable aleatoria, digamos  $x = 1$ , pero por sí misma  $f(1)$  no es significativa en términos de probabilidades para  $X$ . De hecho, dado que  $X$  varía continuamente a lo largo de una parte de los números reales, **la probabilidad de exactamente  $X = 1$  es infinitesimalmente pequeña**. Sin embargo, es **significativo pensar y calcular probabilidades para valores de una variable aleatoria en vecindarios finitos alrededor de  $X = 1$** . La figura 4.3 muestra la probabilidad de que  $X$  esté entre 0,5 y 1,5 como la integral de la PDF entre estos límites.



**FIGURE 4.3** Hypothetical PDF  $f(x)$  for a nonnegative random variable,  $X$ . Evaluation of  $f(x)$  is not, by itself, meaningful in terms of probabilities for specific values of  $X$ . Probabilities are obtained by integrating portions of  $f(x)$ .

# HISTOGRAMAS

## Kernel Density smoothing (KDS)

El Kernel density smoothing es una alternativa al histograma que no requiere redondeo arbitrario a los centros de ubicación y que presenta un resultado uniforme.

Se puede entender el KDS como una extensión del histograma.

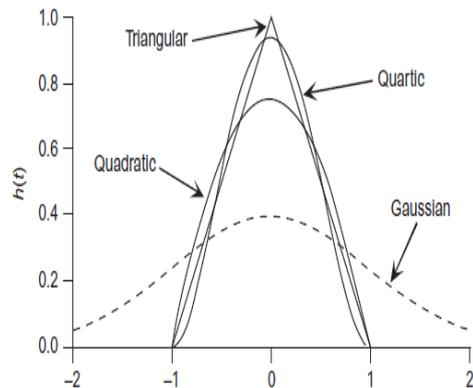
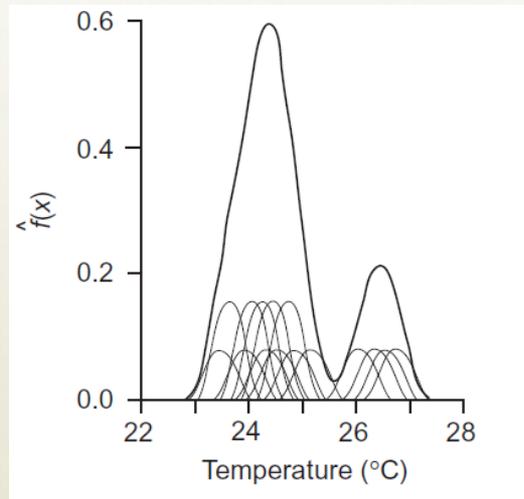
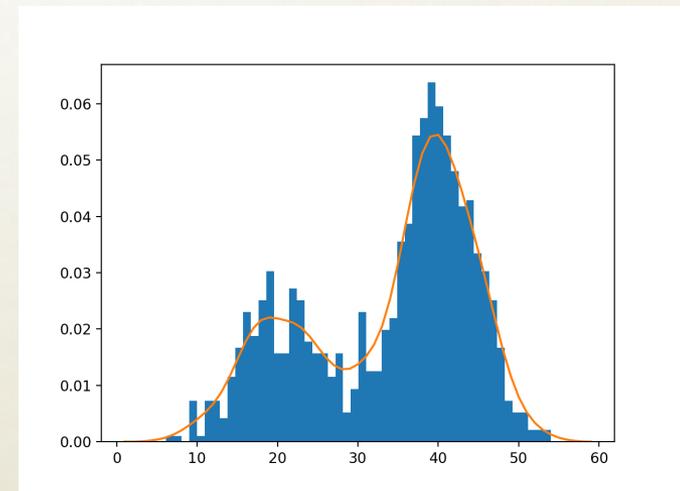


FIGURE 3.7 The four commonly used smoothing kernels defined in Table 3.1.



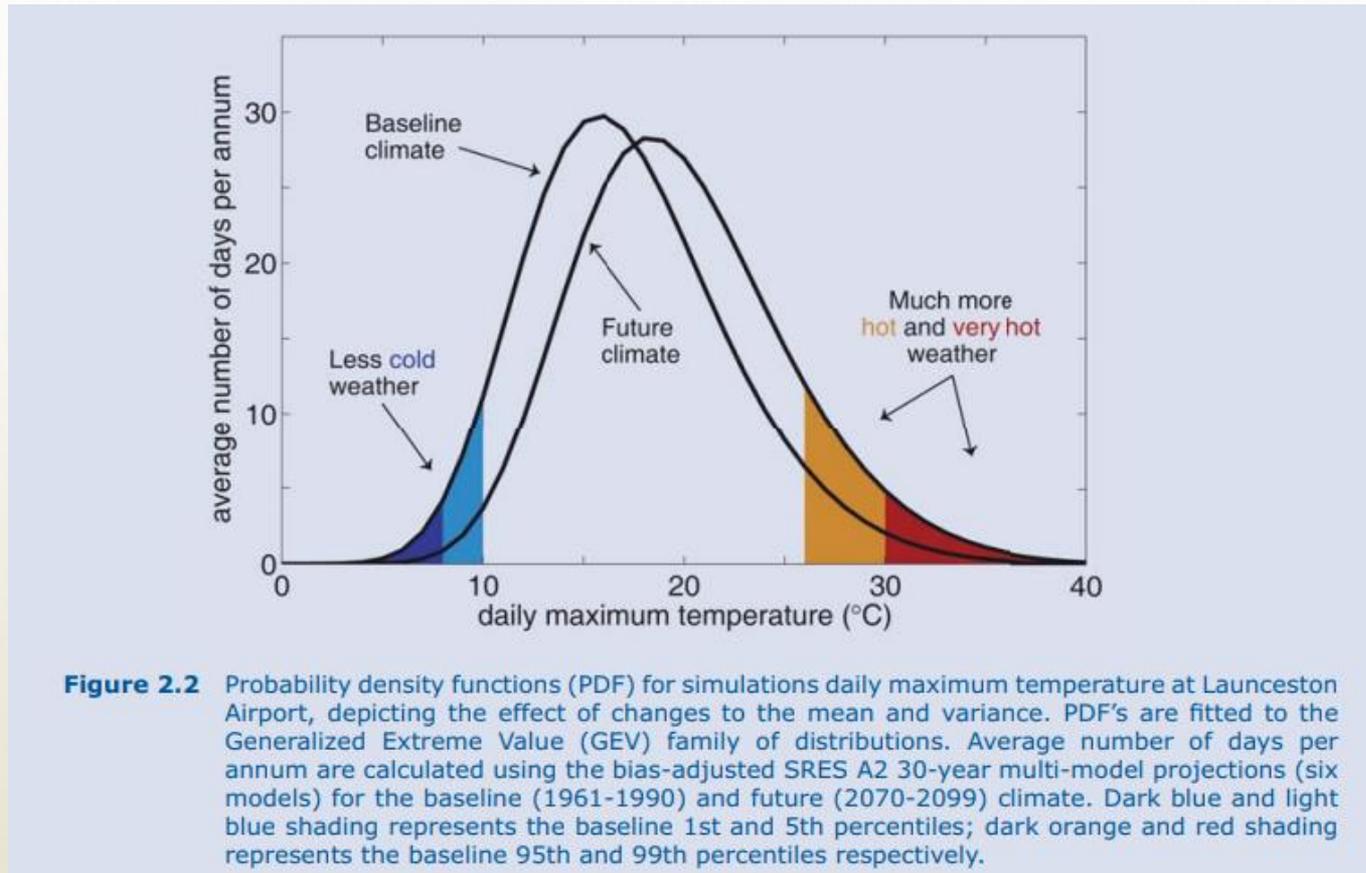
Wilks, 2019



Brownlee, 2019\*

# PDF (PROBABILITY DENSITY FUNCTION)

## Ejemplo:



White et al, 2010

***PRACTICA***

# DISTRIBUCIONES PARAMETRICAS

Distribuciones teóricas, formas matemáticas, que llevan a una idealización de los datos reales.

**Compacidad/compactibilidad:** Una distribución paramétrica bien ajustada reduce el número de cantidades necesarias para caracterizar las propiedades de los datos.

**Suavizado e interpolación:** Los datos reales están sujetos a variaciones de muestreo que conducen a espacios vacíos en sus distribuciones empíricas. Una distribución paramétrica impuesta sobre los datos reales representaría la posibilidad de que ocurran eventos en donde no existe valores en la muestra, además de permitir la estimación de sus probabilidades de ocurrencia.

**Extrapolación:** La estimación de probabilidades para eventos fuera del rango de un conjunto de datos en particular requiere suposiciones sobre el comportamiento aún no observado.

# DISTRIBUCIONES PARAMETRICAS

## **Que es una distribución paramétrica?**

Una distribución paramétrica es una forma matemática abstracta, una forma característica o una familia de formas características. Algunas de estas formas matemáticas surgen naturalmente como consecuencia de ciertos tipos de procesos de generación de datos y, cuando son aplicables, son candidatos especialmente plausibles para representar de manera concisa variaciones en un conjunto de datos. Incluso cuando no existe una fuerte justificación natural detrás de la elección de una distribución paramétrica particular, se puede encontrar empíricamente que la distribución representa muy bien un conjunto de datos.

Wilks, 2019

## **Parámetros vs Estadísticos?**

# DISTRIBUCIONES PARAMETRICAS

## Distribution

- Gaussian
- Lognormal<sup>1</sup>
- Zero-truncated Gaussian
- Logistic
- Gamma
- Exponential
- Chi-square
- Pearson III
- Beta
- Gumbel<sup>2</sup>
- GEV<sup>3</sup>
- Weibull
- Mixed Exponential

Wilks, 2019

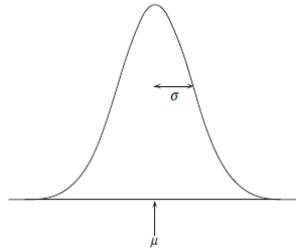


FIGURE 4.4 Probability density function for the Gaussian distribution, Equation 4.24. The mean,  $\mu$ , locates the center of this symmetrical distribution, and the standard deviation,  $\sigma$ , controls the degree to which the distribution spreads out. Nearly all of the probability is within  $\pm 3\sigma$  of the mean.

The PDF for the Gaussian distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], \quad -\infty < x < \infty.$$

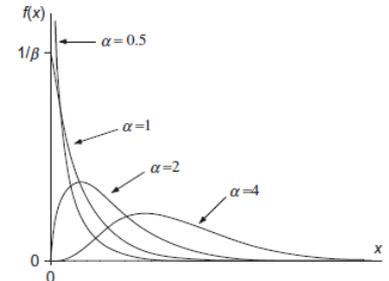


FIGURE 4.9 Gamma distribution density functions for four values of the shape parameter,  $\alpha$ .

The gamma distribution is defined by the PDF

$$f(x) = \frac{(x/\beta)^{\alpha-1} \exp(-x/\beta)}{\beta \Gamma(\alpha)}, \quad x, \alpha, \beta > 0.$$

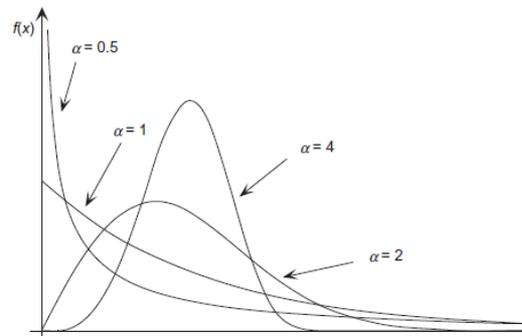
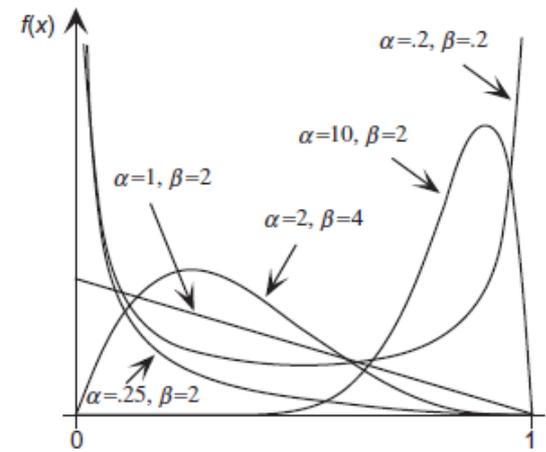


FIGURE 4.13 Weibull distribution PDFs for four values of the shape parameter,  $\alpha$ .

$$f(x) = \left(\frac{\alpha}{\beta}\right) \left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right], \quad x, \alpha, \beta > 0.$$



$$f(x) = \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right] x^{\alpha-1} (1-x)^{\beta-1}, \quad 0 \leq x \leq 1, \alpha, \beta > 0.$$

# TIPOS DE DISTRIBUCIONES PARAMETRICAS

**Table 4.2** Properties of different distributions and the relationship between population parameters and sample statistics

Name of the distribution	Probability mass/distribution function (pmf/pdf)	Cumulative distribution function (CDF)	Range/support	Population parameters and sample statistics
Binomial distribution	$p_x(x) = {}^n C_x p^x (1-p)^{n-x}$	$F_x(x) = \sum_{i=0}^x p_x(i)$	$x = 0, 1, \dots, n$	$\mu = np$ $\sigma^2 = np(1-p)$
Negative binomial distribution	$p_x(x) = {}^{x-1} C_{j-1} p^j (1-p)^{x-j}$	$F_x(x) = \sum_{i=j}^x p_x(i)$	$x = j, j+1, \dots$	$\mu = j/p$ $\sigma^2 = j(1-p)/p^2$
Multinomial distribution	$p_x(x_1, \dots, x_k) = \frac{n!}{x_1! \dots x_k!} p_1^{x_1} \dots p_k^{x_k}$	$F_x(x) = \sum_{i=0}^x p_x(i)$	$x_i = 0, 1, \dots, n$	$\mu = np_i$ $\sigma^2 = np_i(1-p_i)$
Hypergeometric distribution	$p_x(x) = \frac{{}^k C_x \times {}^{N-k} C_{n-x}}{{}^N C_n}$	$F_x(x) = \sum_{i=\max(0, n+k-N)}^x p_x(i)$	$x = \max(0, n+k-N), \dots, \min(n, k)$	$\mu = nk/N$ $\sigma^2 = \frac{nk(N-k)(N-n)}{N^2(N-1)}$
Geometric distribution	$p_x(x) = p(1-p)^{x-1}$	$F_x(x) = 1 - (1-p)^x$	$x = 1, 2, \dots, n$	$\mu = 1/p$ $\sigma^2 = \frac{(1-p)}{p^2}$
Poisson distribution	$p_x(x) = \lambda^x \frac{e^{-\lambda}}{x!}$	$F_x(x) = \sum_{i=0}^x p_x(i)$	$x = 0, 1, \dots$	$\bar{x} = \lambda$ $S_x^2 = \lambda$
Uniform distribution	$f_x(x) = \frac{1}{\beta-\alpha}$	$F_x(x) = \frac{x-\alpha}{\beta-\alpha}$	$\alpha \leq x \leq \beta$	$\mu = \frac{(\alpha+\beta)}{2}$ $\sigma^2 = \frac{(\beta-\alpha)^2}{12}$
Exponential distribution	$f_x(x) = \lambda e^{-\lambda x}$	$F_x(x) = 1 - e^{-\lambda x}$	$x \geq 0$	$\bar{x} = \frac{1}{\lambda}$ $S_x^2 = \frac{1}{\lambda^2}$
Normal distribution	$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$F_x(x) = \int_{-\infty}^x f_x(x) dx$	$-\infty < x < \infty$	$\mu = \bar{x}$ $\sigma = S_x$
Lognormal distribution	$f_x(x) = \frac{1}{x\sqrt{2\pi\beta^2}} e^{-\frac{(\ln x - \alpha)^2}{2\beta^2}}$	$F_x(x) = \int_0^x f_x(x) dx$	$x > 0$	$\bar{y} = \alpha$ $S_y = \beta$ where $y = \ln x$

(continued)

# TIPOS DE DISTRIBUCIONES PARAMETRICAS

Table 4.2 (continued)

Name of the distribution	Probability mass/distribution function (pmf/pdf)	Cumulative distribution function (CDF)	Range/support	Population parameters and sample statistics
Gamma distribution	$f_x(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$	$F_x(x) = \int_0^x f_x(x) dx$	$x \geq 0$	$\bar{x} = \alpha\beta$ $S_x^2 = \alpha\beta^2$
Extreme value type I (Gumbel) distribution	$f_x(x) = \frac{1}{\alpha} \exp\left[\mp \frac{x-\beta}{\alpha} - \exp\left(\mp \frac{x-\beta}{\alpha}\right)\right]$	$F_x(x) = \exp\left(\mp \exp\left(\mp \frac{x-\beta}{\alpha}\right)\right)$	$-\infty < x < \infty$	$\alpha = \frac{\sqrt{6}S_x}{\pi}$ $\beta = \bar{x} - 0.5772\alpha$
Extreme value type III (Weibull) distribution	$f_x(x) = \alpha x^{\alpha-1} \beta^{-\alpha} \exp[-(x/\beta)^\alpha]$	$F_x(x) = 1 - \exp[-(x/\beta)^\alpha]$	$x \geq 0$	$\beta = \bar{x} / [\Gamma(1+1/\alpha)]$ $\frac{[\Gamma(1+2/\alpha) - \Gamma^2(1+1/\alpha)]}{[\Gamma(1+1/\alpha)]^2} = \frac{S_x}{\bar{x}^2}$
Beta distribution	$f_x(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$	$F_x(x) = \int_0^x f_x(x) dx$	$0 < x < 1$	$\bar{x} = \frac{\alpha}{(\alpha+\beta)}$ $S_x^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Pearson type III distribution	$f_x(x) = \frac{\lambda^\beta (x-\varepsilon)^{\beta-1} e^{-\lambda(x-\varepsilon)}}{\Gamma(\beta)}$	$F_x(x) = \int_\varepsilon^x f_x(x) dx$	$x \geq \varepsilon$	$\lambda = \frac{S_x}{\sqrt{\beta}}, \beta = \left(\frac{2}{C_s}\right)^2$ $\varepsilon = \bar{x} - S_x \sqrt{\beta}$
Log-Pearson type III distribution	$f_x(x) = \frac{\lambda^\beta (y-\varepsilon)^{\beta-1} e^{-\lambda(y-\varepsilon)}}{\Gamma(\beta)}$ where $y = \ln x$	$F_x(x) = \int_{e^\varepsilon}^x f_x(x) dx$	$\ln x \geq \varepsilon$	$\lambda = \frac{S_y}{\sqrt{\beta}}, \beta = \left[\frac{2}{C_s(y)}\right]^2$ $\varepsilon = \bar{y} - S_y \sqrt{\beta}$
Chi-square distribution	$f_{\chi^2}(x) = \frac{x^{-(1-\nu/2)} e^{-x/2}}{2^{\nu/2} \Gamma(\nu/2)}$	$F_{\chi^2}(x) = \int_0^x f_{\chi^2}(x) dx$	$x > 0$	$\mu = \nu$ $\sigma^2 = \nu^2$
t-distribution	$f_t(t) = \frac{\Gamma[(\nu+1)/2] (1+t^2/\nu)^{-(\nu+1)/2}}{[\sqrt{\pi\nu} \Gamma(\nu/2)]}$	$F_t(x) = \int_{-\infty}^x f_t(x) dx$	$-\infty < t < \infty$	$\mu = 0$ $\sigma^2 = \frac{\nu}{\nu-2}$
F-distribution	$f_f(x) = \frac{\Gamma[(\nu_1+\nu_2)/2] \nu_1^{\nu_1/2} \nu_2^{\nu_2/2} x^{\nu_1/2-1} (\nu_2+\nu_1 x)^{-(\nu_1+\nu_2)/2}}{[\Gamma(\nu_1/2) \Gamma(\nu_2/2)]}$	$F_f(x) = \int_0^x f_f(x) dx$	$x > 0$	$\mu = \frac{\nu_2}{(\nu_2-2)}$ $\sigma^2 = \frac{\nu_2^2(\nu_1+2)}{[\nu_1(\nu_2-2)(\nu_2-4)]}$

# EVALUACIONES CUALITATIVAS DE LA BONDAD DE AJUSTE

## Superposición de una distribución paramétrica ajustada y un histograma de datos

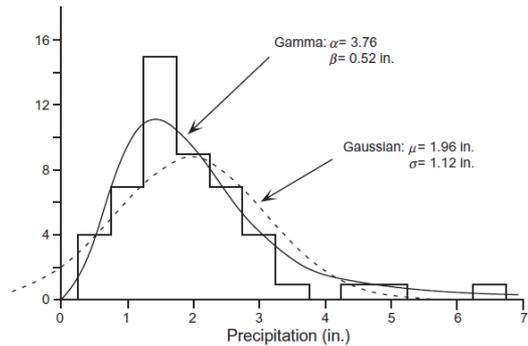


FIGURE 4.16 Histogram of the 1933–1982 Ithaca January precipitation data from Table A.2, with the fitted gamma (solid) and Gaussian (dashed) PDFs. Each of the two density functions has been multiplied by  $A = 25$ , since the bin width is 0.5 in. and there are 50 observations. Apparently the gamma distribution provides a reasonable representation of the data. The Gaussian distribution underrepresents the right tail and implies nonzero probability for negative precipitation.

## Quantile–Quantile (Q–Q) Plots

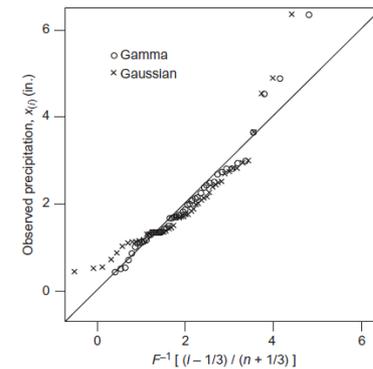
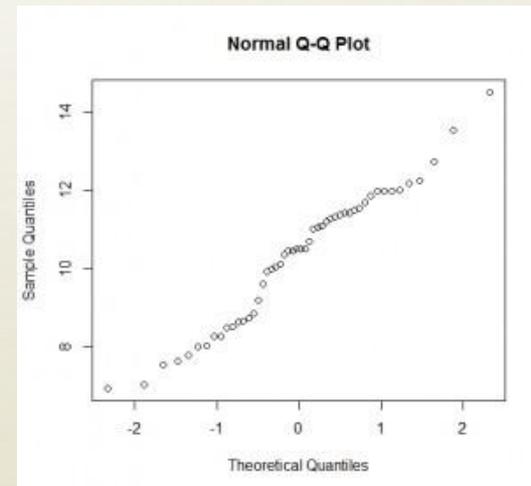


FIGURE 4.17 Quantile–quantile plots for gamma (o) and Gaussian (x) fits to the 1933–1982 Ithaca January precipitation data in Table A.2. Observed precipitation amounts are on the vertical, and amounts inferred from the fitted distributions using the Tukey plotting position are on the horizontal. Diagonal line indicates 1:1 correspondence.

Wilks, 2019



# EVALUACIONES CUANTITATIVAS DE LA BONDAD DE AJUSTE

## Kolmogorov-Smirnov (K-S) Test

### 3.2.1. Kolmogorov-Smirnov (K-S) Test

The K-S test statistic, the *supreme* class of EDF statistics, is based on the maximum vertical difference between the theoretical and empirical distributions [25]. The main goal of this test is to compare the empirical cumulative frequency ( $S_n(x)$ ) with the cdf of an assumed theoretical distribution ( $F_X(x)$ ). The maximum difference between  $S_n(x)$  and  $F_X(x)$  is the K-S test statistic. For a sample size  $n$ , the data is rearranged in increasing order,  $X_1 < X_2 < \dots < X_n$  and the K-S statistic is assessed for each ordered value:

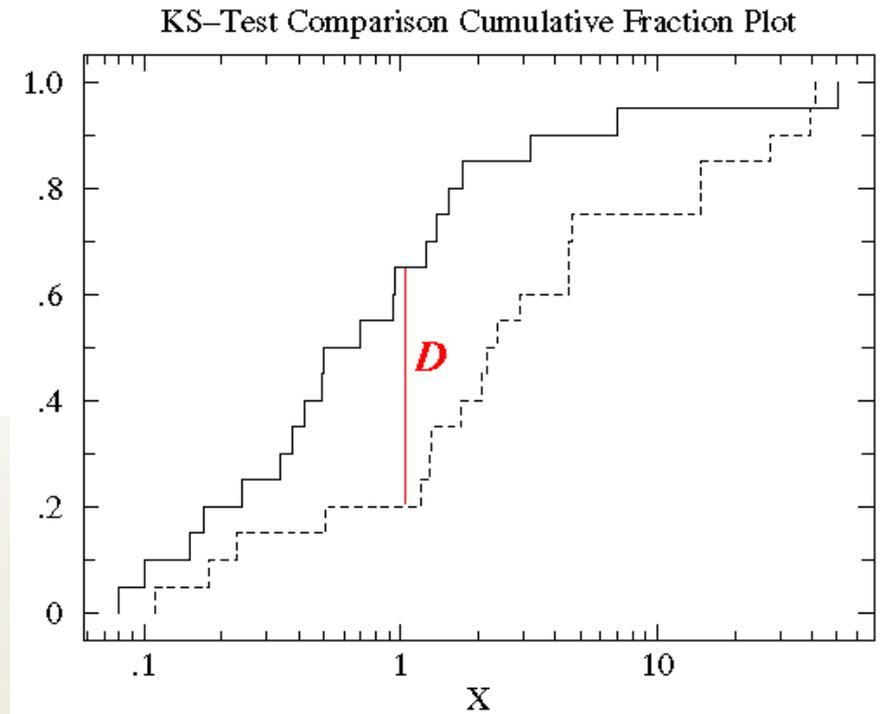
$$\begin{aligned} S_n(x) &= 0; \text{ if } X < X_1 \\ &= k/n; \text{ if } X_k \leq X < X_{k+1} \\ &= 1; \text{ if } X > X_n \end{aligned} \quad (39)$$

$$D_n = \max|F_x(x) - S_n(x)| \quad (40)$$

$$P(D_n \leq D_n^\alpha) = 1 - \alpha \quad (41)$$

where  $D_n^\alpha$  is the critical value,  $\alpha$  is the significance level and  $k$  is the rank order of the data set.

Ashraful, 2017



[https://cxc.harvard.edu/csc/why/ks\\_test.html](https://cxc.harvard.edu/csc/why/ks_test.html)

# EVALUACIONES CUANTITATIVAS DE LA BONDAD DE AJUSTE

## RMSE

### 3.2.3. Root Mean Square Error (RMSE)

The smallest RMSE value indicates the best-fit model of the variate and gives the standard deviation of the model prediction error. It measures the differences between observed and estimated values. The RMSE can be expressed as the following equation:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{j=1}^n (x_i - X)^2} \quad (43)$$

where  $x_i$  denotes the estimated value and  $X$  denotes the observed value. The RMSE gives a relatively larger weight to large errors by squaring them.

Ashraful, 2017

## Anderson-Darling Test

### 3.2.2. Anderson-Darling (A-D) Test

The A-D test, the most powerful EDF test, was first introduced by Anderson & Darling [26] to place more weight at the tails of the distribution [27]. In cases with relatively large extremes, it may be expected the A-D test to be more suitable to select the best-fitted model to data maxima. The A-D test statistic, the *quadratic* class of the EDF test statistic, is expressed as  $A^2$  as follows:

$$A^2 = - \sum_{i=1}^n [(2i-1) \ln F_X(x_i) + \ln[1 - F_X(x_{n+1-i})]] / n - n \quad (42)$$

where  $F_x(x_i)$  is the cdf of the proposed distribution at  $x_i$ , for  $i = 1, 2, \dots, n$ . The observed data must be arranged in increasing order, as  $x_1 < x_2 < \dots < x_n$ .

In the K-S test, both the theoretical and empirical cdfs are relatively flat at the tails of the probability distributions. On the other hand, the A-D test gives more weight to the tails. This can be a more accurate test when the tails of the selected theoretical distribution are the focus of the analysis, as with extreme rainfall.

Ashraful, 2017

# EJERCICIO: PERIODO DE RETORNO

## DEFINICION

(también llamado intervalo de recurrencia). el tiempo promedio hasta la próxima ocurrencia de un evento definido.

cuando el tiempo hasta la siguiente ocurrencia tiene una distribución geométrica, el período de retorno es igual a la inversa de la probabilidad de que el evento ocurra en el siguiente período de tiempo, es decir,  $T = 1 / P$ , donde T es el período de retorno, en número de intervalos de tiempo, y P es la probabilidad de que ocurra el próximo evento en un intervalo de tiempo dado.

AMS

El concepto de período de retorno (también conocido como 'intervalo de recurrencia promedio' o 'intervalo de repetición') de **cualquier evento hidrológico/meteorológico** (por ejemplo, inundaciones, lluvias, descargas de ríos, deslizamientos de tierra, tormentas de viento, tornados). Juega un papel clave en el riesgo y la incertidumbre. análisis en estudios hidroclimáticos. El período de retorno se puede definir como el tiempo promedio para que un evento de magnitud dada sea igualado o superado en un sentido estadístico. Básicamente, es una medición estadística basada típicamente en datos históricos que denotan el intervalo de recurrencia promedio de un evento durante un período de tiempo prolongado.

Maity, 2018

# EJERCICIO: PERIODO DE RETORNO

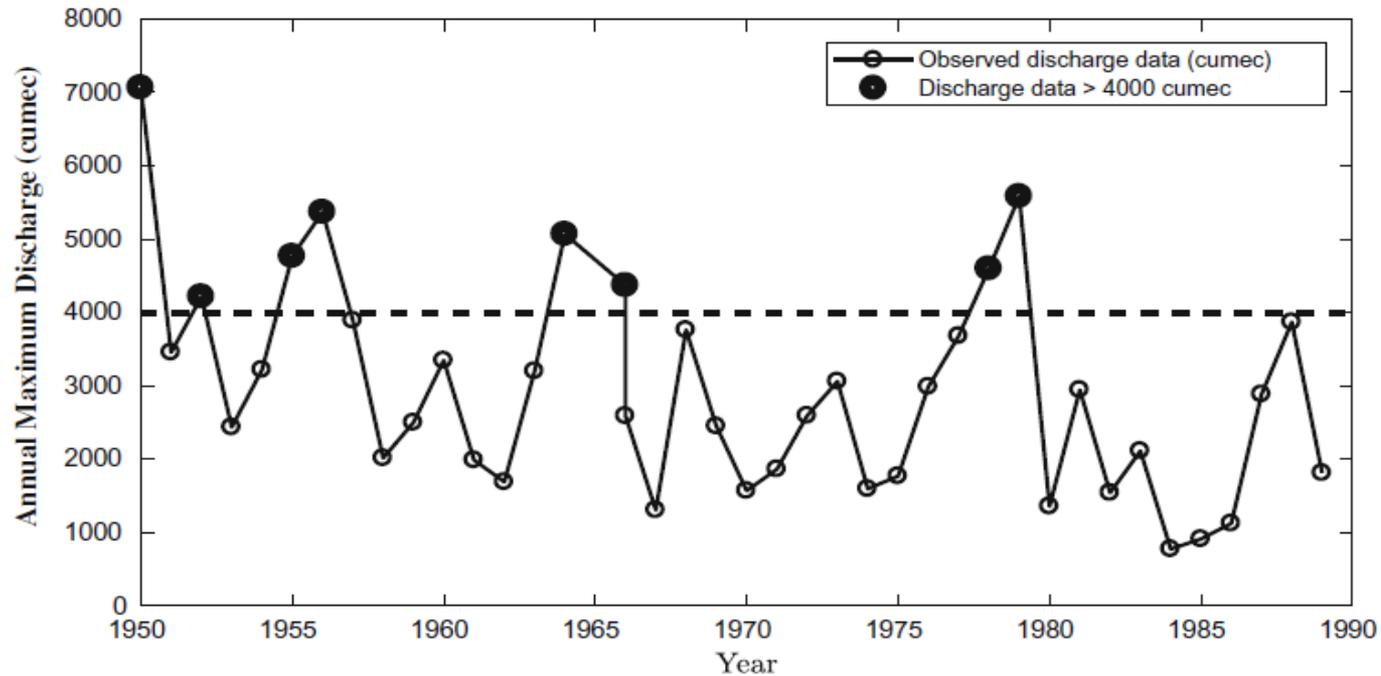


Fig. 5.1 Time series of annual maximum discharges at a river gauging station

# EJERCICIO: PERIODO DE RETORNO



Article

## Best-Fit Probability Distributions and Return Periods for Maximum Monthly Rainfall in Bangladesh

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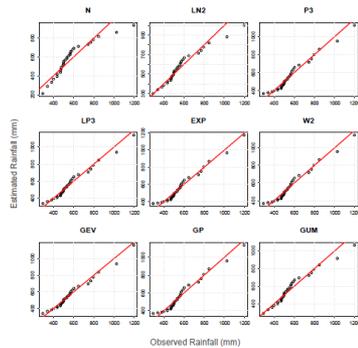


Figure 2. Q-Q plot of all distributions at Station: Dinajpur.

Sl.no	Station Name	Mean ( $\bar{X}$ )	SD ( $\sigma$ )	Coeff. of Skew. ( $\gamma$ )	Best-Fit Test Statistic Results			Highest Ranked Distribution (Sum of Ranks)
					By K-S Test	By A-D Test	By RMSE Test	
1	Ambagan	875.2	209.2	0.118	LN2(0.115)	P3(0.305)	N(38.25)	N(5)
2	Barisal	555.0	167.6	1.618	GEV(0.095)	GEV(0.382)	GEV(37.95)	GEV(3)
3	Bhola	600.6	174.8	0.675	GUM(0.081)	LP3(0.280)	P3(28.71)	P3(7)
4	Bogra	483.9	151.1	0.187	LP3(0.066)	LP3(0.213)	LP3(21.05)	LP3(3)
5	Chandpur	549.8	215.3	2.102	GEV(0.088)	GEV(0.136)	LP3(27.51)	GEV(4)
6	Chittagong	897.4	238.0	0.335	GP(0.068)	GP(0.249)	GP(28.14)	GP(3)
7	Chuadanga	433.2	147.8	0.923	P3(0.07)	LN2(0.18)	GUM(16.18)	GEV(7)
8	Comilla	531.2	152.0	0.656	GP(0.082)	P3(0.243)	W2(18.53)	P3(5)
9	Cox's Bazar	1116.9	243.1	0.840	P3(0.108)	P3(0.731)	LN2(67.98)	P3 & LP3(7)
10	Dhaka	511.3	147.1	0.296	LN2(0.098)	LP3(0.326)	W2(20.04)	LP3(6)
11	Dinajpur	574.1	194.6	1.592	GEV(0.102)	GEV(0.432)	GEV(28.82)	GEV(3)
12	Faridpur	450.3	156.2	1.106	GEV(0.083)	GEV(0.233)	P3(27.34)	GEV(7)
13	Feni	791.6	182.8	0.383	GUM(0.113)	P3(0.453)	P3(36.07)	P3(6)
14	Hatiya	858.5	249.7	-0.112	GUM(0.062)	P3(0.226)	N(36.49)	GEV(6)
15	Ishurdi	402.1	134.2	0.433	LN2(0.07)	LP3(0.19)	GP(16.19)	P3 & LP3(9)
16	Jessore	460.4	170.9	1.103	GEV(0.071)	LP3(0.181)	P3(22.27)	LP3(7)
17	Khepupara	766.9	150.7	0.158	GEV(0.075)	GEV(0.263)	GP(19.3)	GEV(5)
18	Khulna	483.2	159.2	0.778	P3(0.066)	P3(0.193)	P3(23.71)	P3(3)
19	Kutubdia	930.9	293.4	1.514	GEV(0.08)	GEV(0.382)	LP3(48.937)	GEV(4)
20	Madaripur	518.6	154.1	0.580	P3(0.07)	LP3(0.18)	W2(18.32)	P3(5)
21	Majidi Court	807.5	202.2	1.139	GEV(0.066)	GUM(0.25)	GEV(31.24)	GEV(4)
22	Mongla	484.9	143.7	1.929	LP3(0.08)	LP3(0.219)	GEV(32.543)	LP3 & GEV(5)
23	Mymensingh	558.8	164.3	0.142	GP(0.105)	GP(0.36)	GP(25.12)	GP(3)
24	Pataakhali	707.8	175.6	0.156	P3(0.082)	P3(0.236)	P3(22.59)	P3(3)
25	Rajshahi	388.0	123.6	1.107	LN2(0.084)	P3(0.198)	LP3(18.78)	LP3(7)
26	Rangamati	705.1	206.8	0.450	GUM(0.064)	GEV(0.262)	LP3(34.64)	GEV(7)
27	Rangpur	634.0	202.6	1.483	GP(0.067)	P3(0.192)	LP3(22.08)	LP3(7)
28	Sandwip	1096.9	496.5	2.399	GEV(0.054)	GEV(0.125)	LP3(66.31)	LP3(5)
29	Satkhira	449.6	117.6	0.453	W2(0.091)	P3(0.306)	P3(22.27)	P3(7)
30	Sitakunda	890.1	240.6	0.244	P3(0.076)	GEV(0.274)	GP(29.48)	GEV(6)
31	Srimangal	596.2	176.6	0.937	P3(0.05)	LP3(0.135)	GUM(17.37)	GEV & LP3(7)
32	Sydpur	625.8	171.4	-0.103	LN2(0.104)	N(0.212)	P3(21.5)	N(5)
33	Sylhet	979.8	206.6	0.246	GEV(0.06)	GEV(0.215)	GP(18.68)	GEV(6)
34	Tangail	444.8	125.1	0.930	P3(0.066)	GEV(0.13)	P3(12.77)	P3(7)
35	Teknaf	1332.4	238.6	1.022	GUM(0.065)	LP3(0.161)	GUM(29.63)	GUM(6)

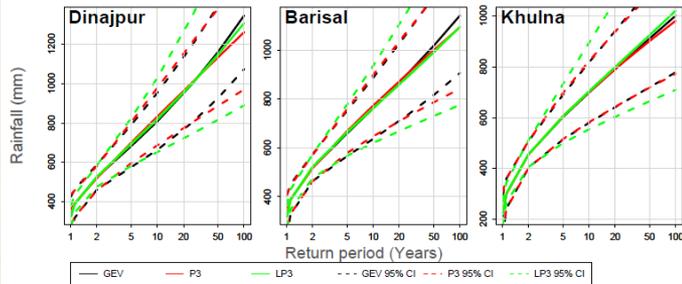


Figure 4. Rainfall return level estimations (solid lines) and 95% confidence intervals (dashed lines) of the top three fitted distributions—GEV, P3 and LP3 of three stations, as an example.

***PRACTICA***